Foreshadowing elastic instabilities by negative group velocity in soft composites – Supplementary Materials

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In the main part of the manuscript, we demonstrated how the analysis of S-wave propagation in LCs subjected to finite in-plane deformation with the following homogeneous macroscopic deformation gradient

\[ \mathbf{F}_{ps} = \lambda^{-1} \mathbf{e}_1 \otimes \mathbf{e}_1 + \lambda \mathbf{e}_2 \otimes \mathbf{e}_2 + \mathbf{e}_3 \otimes \mathbf{e}_3, \]

(1)

can be used to predict elastic instabilities. However, our findings can be easily expanded on the layered and fibrous composites subjected to uniaxial deformation in fully 3D formulation. The corresponding macroscopic deformation gradient takes form

\[ \mathbf{F}_{ut} = \lambda \mathbf{e}_2 \otimes \mathbf{e}_2 + \lambda^{-1/2} (\mathbf{I} - \mathbf{e}_2 \otimes \mathbf{e}_2). \]

(2)

First, we consider the in-plane S-wave propagation along the layers in the uniaxially deformed LCs. FIG. SM-1 presents the frequency ((a) and (d)), the phase velocity ((b) and (e)), and the group velocity ((c) and (f)) as functions of the wavenumber for the in-plane S-wave propagating along the layers in the LCs undergoing instabilities via macroscopic (first row) and microscopic (second row) scenarios. Comparison of FIGs. 2, 3 of the main text and FIG. SM-1 shows that 3D contraction of the LCs influences S-wave propagation similarly to the 2D contraction. For instance, similarly to the 2D contraction, the uniaxial deformation barely influences phase velocities of the short waves in LCs undergoing macroscopic loss of stability. In particular, the phase velocities of the long S-waves significantly decrease in the contracted LC (see FIG. 2 (b)). However, the contraction barely influences phase velocities of the short S-waves with the wavelengths being slightly lower than the period of the LC \((l \lesssim d)\). The opposite effect is observed when the shear modulus contrast and geometrical parameters dictate the microscopic buckling: the most affected S-waves in this case are shorter than the period of LC. Generally speaking, LCs subjected to uniaxial (2) or in-plane (1) contractions have very similar behavior in respect of their stability. For instance, the critical wavelengths are almost the same for these two compression modes\(^1\). Moreover, similarly to the case, considered in the main part of the Letter, NGV is observed in the vicinity of elastic instability only if the geometrical characteristics and material properties of the constituents dictate the microscopic buckling (FIG. SM-1(f)).

Second, we analyze the S-wave propagation along the fibers in the periodic fibrous composites (FCs) with a square arrangement of fibers. Geometrically, the fibers are characterized by their diameters, namely \(D = 2a \sqrt{v_f / \pi} \), where \(v_f < \pi / 4\) is the volume fraction of the
Figure SM-1. Frequency ((a) and (d)), phase velocity ((b) and (e)), and group velocity ((c) and (f)) curves as functions of wave number for the in-plane S-waves propagating along the layers in LCs with $\mu_l/\mu_m = 15$, $v_l = 0.2$ (first row: (a), (b), and (c)) and $v_l = 0.02$ (second row: (d), (e), and (f)). The LCs are subjected to the uniaxial contractions with the various stretch ratios. The corresponding critical stretch ratios are $\lambda_{cr}^{\text{macro}} = 0.878$ and $\lambda_{cr}^{\text{micro}} = 0.847$ for $v_f = 0.2$ and $v_f = 0.02$, respectively.

fibers and $a$ is the period of the FC (see Fig. 1 (b)). The phase velocities of both S-waves propagating along the fibers in the uniaxially deformed FC coincide. Therefore, the phase velocities of both S-waves become zero in the FC subjected to the critical stretch, whereas in the LC experiencing the critical deformation only the phase velocity of the in-plane S-wave becomes zero. FIG. SM-2 shows the frequency ((a) and (d)), the phase velocity ((b) and (e)), and the group velocity ((c) and (f)) as functions of the wavenumber for the S-waves propagating along the fibers in the FCs experiencing instabilities via macroscopic (first row) and microscopic (second row) scenarios. The FCs undergo the uniaxial contraction with the macroscopic deformation gradient (2). The critical stretch ratios are $\lambda_{cr}^{\text{macro}} = 0.856$ and $\lambda_{cr}^{\text{micro}} = 0.733$ for $v_f = 0.2$ and $v_f = 0.02$, respectively. Consistently with the previous observations for the LC exhibiting an instability via the macroscopic mechanism, the deformation of the FC with 20% of fibers mostly influences the shear waves with wavelengths.
Figure SM-2. Frequency ((a) and (d)), phase velocity ((b) and (e)), and group velocity ((c) and (f)) curves as functions of wavenumber for the S-waves propagating along the fibers in FCs with $\mu_f/\mu_m = 15$, $v_f = 0.2$ ((a)–(c)) and $v_f = 0.02$ ((d)–(f)). The FCs are subjected to the uniaxial contractions with the various stretch ratios. The corresponding critical stretch ratios are $\lambda_{\text{macro}}^{\text{cr}} = 0.856$ and $\lambda_{\text{micro}}^{\text{cr}} = 0.733$ for $v_f = 0.2$ and $v_f = 0.02$, respectively. Frequency is normalized as $f^* = \frac{\omega}{2\pi} \sqrt{\rho/\tilde{\mu}}$; velocities are normalized by $v_0 = \sqrt{\tilde{\mu}/\rho}$, where $\tilde{\mu} = \mu_m((1 + v_f)\mu_f + v_m\mu_m)/(v_m\mu_f + (1 + v_f)\mu_m)$.

larger than the period of the FC, namely $l \gtrsim a$ (see FIGs. SM-2 (a)–(c)). In particular, the uniaxial contraction of $\lambda = 0.86$ decreases the phase and the group velocities of the long S-waves ($l \gg a$) from $v_0$ down to 0.19$v_0$, while it decreases the phase velocity from 1.17$v_0$ down to 0.79$v_0$ and increases the group velocity from 1.21$v_0$ up to 1.32$v_0$ for S-waves with wavelength $l = 2a$ (see FIGs. SM-2 (b)-(c)). Note that the phase and group velocities of S-waves are maximal for certain wavelengths depending on the applied deformation; for example, $v_{\text{gr}}^{\text{max}} = 1.33v_0$ for the S-wave with wavelength $l = 1.87a$ propagating along the fibers in the FC contracted to $\lambda = 0.86$, while $v_{\text{gr}}^{\text{max}} = 1.29v_0$ for S-wave with wavelength $l = 2.63a$ propagating along the fibers in the undeformed FC (see FIG. SM-2 (c)). At the same time, the uniaxial compression of the FC with 2% of fibers close to the onset of instability drastically influences the S-waves with the moderate wavelengths, namely
0.4a ≲ l ≲ 2a (see FIGs. SM-2 (d)–(f)). In particular, the uniaxial contraction of \( \lambda = 0.734 \) decreases the phase velocity of S-wave with wavelengths \( l = 0.73a \) from 1.01\( v_0 \) down to 0.08\( v_0 \) (see FIG. SM-2 (e)). Furthermore, the dispersion curve of the S-wave propagating in FC contracted to \( \lambda = 0.734 \) has negative slope for certain range of wavelengths, namely \( 0.74a ≲ l ≲ 1.13a \) (see FIG. SM-2 (d)); consequently, the group velocity of the S-wave for this wavelength range is negative or antiparallel to the phase velocity. However, the uniaxial compression close to the unstable state also increases the group velocities of S-waves with certain wavelengths; for example, the group velocity of S-wave with \( l = 0.51a \) increases from \( v_{gr} = 0.98v_0 \) up to \( v_{gr} = 1.80v_0 \) in the FC compressed to \( \lambda = 0.734 \) (see FIG. SM-2 (f)). Since the wavenumber corresponding to the onset of microscopic instability depends on the volume fraction of the stiffer phase and contrast in shear moduli in the layered and fibrous composites\(^1\), we can predetermine the most affected by deformation range of wavelengths through the proper choice of the material composition.

Finally, we conclude by saying that layered and fibrous composites demonstrate almost the identical behavior in the vicinity of the microscopic instability. For all considered cases, there is a stretch range, for which the group velocity is antiparallel to the phase velocity regardless of the propagation direction. Reversible transition between states with positive and negative group velocities can be applicable for the predicting the oncoming buckling and tuning of the acoustic properties in materials and structures.

REFERENCES