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Elastic Wave Propagation in Soft Microstructured Composites Undergoing Finite Deformations

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We consider the propagation of elastic waves in soft composite materials undergoing large deformations. The analysis is performed in terms of small amplitude motions superimposed on a deformed state. By consideration of 2D periodic laminates and 3D fiber composites, we find that an applied deformation influences the elastic waves through the change in the microstructure, and through the change in the local material properties. These effects can be significantly amplified by the deformation induced elastic instability phenomenon leading to microstructure transformations.

1 Introduction

Elastic wave propagation has been investigated intensively because the understanding of this phenomenon is important for a large variety of applications. The fields of applications include nondestructive material testing, ultrasonic transducers, vibration dumpers, wave-guides, acoustic mirrors and filters. Recently, a new class of synthetic acoustic metamaterials has attracted a considerable attention. The attractive properties of these metamaterials originate in their microstructure, which can be predesigned to significantly affect elastic wave propagation.

Furthermore, soft metamaterials, due to their capability to undergo finite deformations, open the promising opportunities of tuning elastic/acoustic properties via deformation [1–4]. Imposed deformations influence the wave propagation in two ways: first, the geometry of the microstructure evolves with the deformation; second, local deformations that the material experiences lead to inhomogeneity in local material properties (such as local stiffening [2]). Soft biological tissues often can be found in similar, pre-stressed or pre-deformed conditions due to, for example, growth, damage or remodeling. This condition can be used to trigger sudden and reversible pattern transformations which are frequently accompanied by changes in elastic/acoustic properties. Here, we specially focus on the influence of instability-induced interfaces on elastic wave propagation in finitely deformed layered materials. It is known that layered materials compressed along the layers can demonstrate bifurcations [5–9]. This phenomenon can be used for inducing wavy interfaces with tunable wavelengths and amplitudes [6].

2 Analysis and Discussion

To take into account finite deformation non-linear effects as well as material non-linearity, we analyze the wave propagation in terms of incremental small-amplitude motions superimposed on a finitely deformed state [2–4]. The linearized equation of motion can be written as

$$\mathcal{A}_{0ijkl} v_{k,lj} = \rho \frac{\partial^2 v_i}{\partial t^2}, \tag{1}$$

where $\mathcal{A}_{0iqkp} = J^{-1} \mathcal{A}_{ijkl} F_{pl} F_{qj}$ and $\rho = J^{-1}\rho_0$ is the density of the deformed material. The tensor of elastic moduli is defined through the material model energy function $\psi(F_{ij})$, such that $\mathcal{A}_{i\alpha k\beta} = \partial^2\psi/\partial F_{i\alpha}\partial F_{k\beta}$. Note that the tensor of elastic moduli $\mathcal{A}_{i\alpha k\beta}$ is a function of the deformation gradient, in general. If an effective energy function for a composite is available (for example, the effective energy function for laminates or transversely isotropic fiber composite [5]), the long wave estimates for plane waves can be derived directly from Eq. (1). To obtain the long wave estimate, we seek a solution for Eq. (1) in the form of plane waves with constant polarization. This substitution leads to an eigenvalue problem allowing to calculate the phase velocities as functions of applied deformation and material composition and consentient properties.

To account for interactions of the elastic waves with the material microstructure, we employ the Bloch-Floquet analysis of waves propagating in periodic media [1]. While usually the Bloch-Floquet technique requires an implementation in a numerical tool such as the finite element method, for layered materials, a closed form expression for the dispersion relation can be obtained. We extended the analysis of elastic waves propagating in the perpendicular direction to the layers and account for the effects of the finite deformation [10]. We find that the propagation of the longitudinal or pressure waves is mostly affected through the change in the geometry; while for the transverse or shear waves the effect of material non-linearity compete with the geometrical change effect. Thus, for example, these two competing effects cancel each other for the composites with

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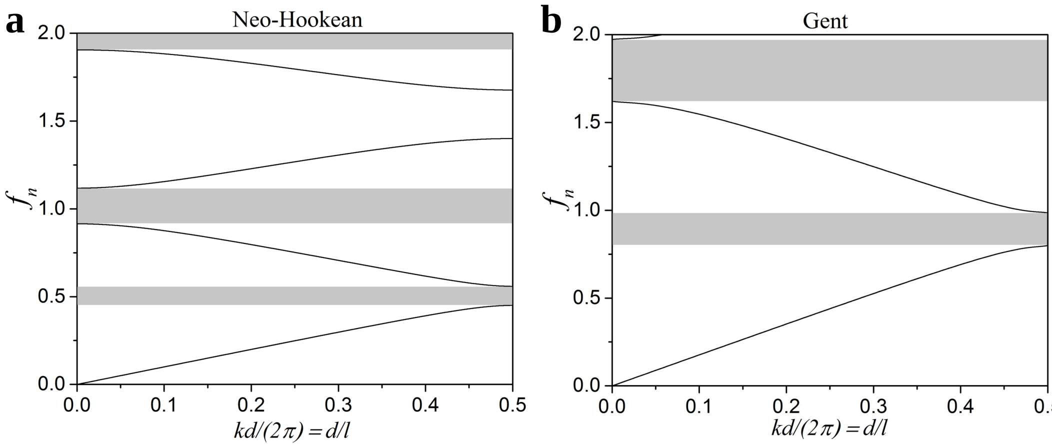

Fig. 1: Dispersion diagram for shear waves propagating perpendicular to the layer in finitely deformed laminates with the volume fraction of the stiffer phase $c^{(2)} = 0.2$, the contrast ratio in the shear moduli $\mu^{(2)}/\mu^{(1)} = 1000$, and the first Lame coefficients $\Lambda^{(2)}/\mu^{(2)} = 10$, $\Lambda^{(1)}/\mu^{(1)} = 100$. (a) – neo-Hookean phases ; (b) – Gent phases with the lock-up parameters $J_m^{(1)} = J_m^{(2)} = 2.5$, with corresponding lock-up stretch ratio $\lambda = 1.64$. Both composites are subjected to the biaxial tension of $\lambda = 1.5$. The grey filled area represents the band-gaps corresponding to frequency ranges at which the waves cannot propagate in the composite.

neo-Hookean phases. However, for the composites with the phases characterized by a significant stiffening effect captured by the Gent material model of the phases, the change in the effective stiffness leads to the significant tunability of the shear waves by deformation. An illustration of the dispersion relations is shown in Fig. 1 for composites with (a) neo-Hookean and (b) Gent phases. The composites with identical densities of the phases are subjected to the biaxial tension of $\lambda = 1.5$. In Fig. (1), the normalized frequency is $f_n = f d^o \sqrt{\rho^{(2)}/\tilde{\mu}}$, where $f = \omega/(2\pi)$, $\rho^{(2)}$ is the density of the matrix, d is the total thickness of the stiffer and softer layers, and $\tilde{\mu} = (c^{(1)}/\mu^{(1)} + c^{(2)}/\mu^{(2)})^{-1}$; here μ is the initial shear moduli and $c^{(2)}$ is the volume fraction of the stiffer phase. We note that the rigorous Bloch wave analysis nicely agrees with the closed form long estimates derived from the homogenized energy functions. The 2D periodic laminates share some similarities with 3D transversely isotropic hyperelastic composites. For example, both composites exhibit the dispersion phenomenon when the elastic waves propagate along the layer/fiber direction [11]. The appearance of the dispersion is governed by the material composition, and can be tuned by applied deformation [11].

The influence of deformation can be further amplified on account the elastic instability phenomenon and the associated microstructure transformations. The onset of instability in initially deformed interfacial layers occurs when a critical compressive strain or stress is achieved [6, 12]. Further compression beyond the critical strain leads to an increase in the wrinkle amplitude of the interfacial layer. This, in turn, gives rise to the formation of a system of periodic scatterers, which interfere with wave propagation. We show that that the topology of wrinkling interfacial layers can be controlled by deformation, and can be further used to produce band-gaps and to filter undesirable frequencies [1]. We find that the band-gap formation mechanism can be achieved even for composites with similar or identical densities. Since the microstructure change is reversible, the method can be used for tuning and manipulating wave propagation by use of deformation in a reversible manner.

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